Chapter 1, 3, and 4 IB Style Review Questions

Sequences and Series

{M2007TZ1P1Q14}

An infinite geometric series is given by $\sum_{k=1}^{\infty} 2(4-3x)^k$.

(a) Find the values of x for which the series has a finite sum.

(b) When $x = 1.2$, find the minimum number of terms needed to give a sum which is greater than 1.328.

QUESTION 14

(a) $r = 4 - 3x \Rightarrow$

$|4-3x| < 1$ *M1*

$-1 < 4-3x < 1$

$1 < x < \frac{5}{3}$ *A1* *N1*

(b) $x = 1.2$

$\Rightarrow a = 0.8 \quad r = 0.4$ *(A1)*

$S_n = \frac{0.8(1-0.4^n)}{0.6}$ *A1*

So $\frac{0.8(1-0.4^n)}{0.6} > 1.328$

Solving gives $n > 6.02$ *(A1)*

7 terms are needed *A1* *N4*

Note: Generating terms of the series to find that 7 terms are needed is an alternative method.

Counting Principles (Combinations and Permutations)

{M2006TZ0P1Q19}

There are 10 seats in a row in a waiting room. There are six people in the room.

(a) In how many different ways can they be seated?

(b) In the group of six people, there are three sisters who must sit next to each other. In how many different ways can the group be seated?

QUESTION 19

(a)	A recognition of a permutation of six from ten in words or symbols	***(M1)***	
	Total number of ways $= 151\,200$	***A1***	***N2***
(b)	Total number of ways $= 8 \times 3! \times 7 \times 6 \times 5$	***A1A1A1***	
	$= 10\,080$	***A1***	***N0***

Note: Award ***A1*** for 8 ***A1*** for 3! and ***A1*** for $7 \times 6 \times 5$.

Binmonial Theorem

{N2005TZ0P2Q4}

[Maximum mark: 10]

(a) Write down the term in x^r in the expansion of $(x+h)^n$, where $0 \le r \le n$, $n \in \mathbb{Z}^+$. *[1 mark]*

(b) Hence differentiate x^n, $n \in \mathbb{Z}^+$, from first principles. *[5 marks]*

(c) Starting from the result $x^n \times x^{-n} = 1$, deduce the derivative of x^{-n}, $n \in \mathbb{Z}^+$. *[4 marks]*

4. (a) r^{th} term $= \begin{pmatrix} n \\ n-r \end{pmatrix} x^r h^{n-r} \left(= \dfrac{n!}{r!(n-r)!} x^r h^{n-r} \right)$ ***(A1)***

[1 mark]

(b) $\dfrac{\mathrm{d}(x^n)}{\mathrm{d}x} = \lim\limits_{h \to 0} \left(\dfrac{(x+h)^n - x^n}{h} \right)$ ***(M1)***

$= \lim\limits_{h \to 0} \left(\dfrac{x^n + \begin{pmatrix} n \\ 1 \end{pmatrix} x^{n-1} h + \begin{pmatrix} n \\ 2 \end{pmatrix} x^{n-2} h^2 + \ldots + h^n - x^n}{h} \right)$ ***(A1)***

$= \lim\limits_{h \to 0} \left(\dfrac{x^n + nx^{n-1} h + \dfrac{n(n-1)}{2} x^{n-2} h^2 + \ldots + h^n - x^n}{h} \right)$ ***(A1)***

$= \lim\limits_{h \to 0} \left(nx^{n-1} + \dfrac{n(n-1)}{2} x^{n-2} h + \ldots + h^{n-1} \right)$ ***(A1)***

Note: Accept first, second and last terms in the 3 lines above.

$= nx^{n-1}$ ***(A1)***

[5 marks]

(c) $x^n \times x^{-n} = 1$

$x^n \dfrac{\mathrm{d}(x^{-n})}{\mathrm{d}x} + x^{-n} \dfrac{\mathrm{d}(x^n)}{\mathrm{d}x} = 0$ ***(M1)***

$x^n \dfrac{\mathrm{d}(x^{-n})}{\mathrm{d}x} + x^{-n} \times nx^{n-1} = 0$ ***(A1)***

$x^n \dfrac{\mathrm{d}(x^{-n})}{\mathrm{d}x} + nx^{-1} = 0$ ***(A1)***

$\dfrac{\mathrm{d}(x^{-n})}{\mathrm{d}x} = \dfrac{-nx^{-1}}{x^n} \left(= -nx^{-(1+n)} \right)$ ***(A1)***

[4 marks]

Total [10 marks]

Math Induction

{N2006TZ0P2Q1B}

Part B *[Maximum mark: 11]*

(a) Use mathematical induction to prove that

$$(1)(1!)+(2)(2!)+(3)(3!)+\ldots+(n)(n!)=(n+1)!-1 \text{ where } n\in\mathbb{Z}^{+}.$$ *[8 marks]*

(b) Find the minimum number of terms of the series for the sum to exceed 10^9. *[3 marks]*

Part B

(a) If $n=1$, then $(1)(1!)=(1+1)!-1$ is true **A1**

Assume true for $n=k$

$\Rightarrow (1)(1!)+(2)(2!)+\ldots+(k)(k!)=(k+1)!-1$ **M1A1**

Add the next term $(k+1)(k+1)!$ to both sides **M1**

$(1)(1!)+(2)(2!)+\ldots+(k)(k!)+(k+1)(k+1)!=(k+1)!-1+(k+1)(k+1)!$ **A1**

$=(k+1)![1+k+1]-1$ **A1**

$=(k+2)!-1$ **A1**

True for $k \Rightarrow$ True for $k+1$ and since true for $n=1$, result proved by mathematical induction. **R1**

[8 marks]

(b) $(n+1)!-1>1000000000$ ***(M1)***

$(n+1)!>1000000001$

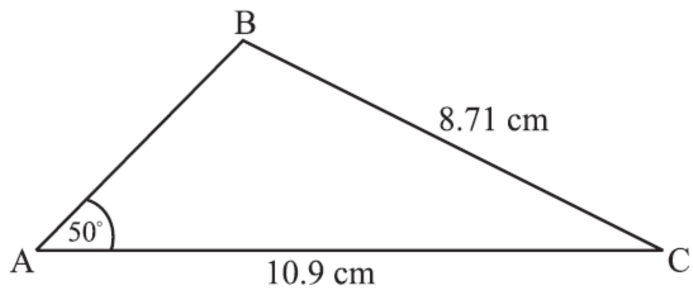
from GDC minimum value of $n=12$ **A2** **N3**

[3 marks]

Sub-total [11 marks]

In the **obtuse-angled** triangle ABC, $\text{AC} = 10.9 \text{ cm}$, $\text{BC} = 8.71 \text{ cm}$ and $\text{B\hat{A}C} = 50^\circ$.

Not to scale



Find the area of triangle ABC.

QUESTION 7

METHOD 1

$$\frac{\sin 50^\circ}{8.71} = \frac{\sin \hat{B}}{10.9}$$ *(M1)*

$\sin \hat{B} = 0.958(65...)$ *(A1)*

Finding the obtuse value of $\hat{B}$ from the range 106 to 107 ***M1***

Finding $\hat{C}$ from the range 23 to 24 *(M1)*

$$\text{Area ABC } = \frac{1}{2} \times 10.9 \times 8.71 \times \sin \hat{C}$$ ***M1***

$= 18.9 \text{ (cm}^2)$ ***A1*** ***N0***

METHOD 2

Using cosine rule *(M1)*

$8.71^2 = \text{AB}^2 + 10.9^2 - 2\text{AB} \times 10.9 \cos 50^\circ$ *(A1)*

Solving a quadratic in AB *(M1)*

choosing $\text{AB} = 4.52(7...)$ ***M1***

$$\text{Area triangle ABC} = \frac{1}{2} \times 10.9 \times AB \sin 50^\circ$$ ***M1***

$= 18.9 \text{ (cm}^2)$ ***A1*** ***N0***

Roots of Polynomial Functions

4. *[Maximum mark: 6]*

The roots of a quadratic equation $2x^2 + 4x - 1 = 0$ are α and β.

Without solving the equation,

(a) find the value of $\alpha^2 + \beta^2$; *[4]*

(b) find a quadratic equation with roots α^2 and β^2. *[2]*

4. (a) using the formulae for the sum and product of roots:

$\alpha + \beta = -2$ ***A1***

$\alpha\beta = -\frac{1}{2}$ ***A1***

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ ***M1***

$= (-2)^2 - 2\left(-\frac{1}{2}\right)$

$= 5$ ***A1***

[4 marks]

Note: Award ***M0*** for attempt to solve quadratic equation.

(b) $(x - \alpha^2)(x - \beta^2) = x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2$ ***M1***

$x^2 - 5x + \left(-\frac{1}{2}\right)^2 = 0$ ***A1***

$x^2 - 5x + \frac{1}{4} = 0$

Note: Final answer must be an equation. Accept alternative correct forms.

[2 marks]

Total [6 marks]